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Quantum Thermal Effect of Arbitrarily Accelerating Kinnersley Black Hole

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The Hawking radiation of an arbitrarily accelerating Kinnersley black hole is studied. We obtain the event horizon equation and the Hawking thermal spectrum formula. Both the location and the temperature of the event horizon depend on the time and the angles. We recover the well-known results when the acceleration vanishes.

1. INTRODUCTION

Recently, we have suggested a new method to determine the location and the temperature of event horizons of nonstationary black holes (Zhao and Dai, 1991, 1992; Zhao and Li, 1993; Yang and Zhao, 1993). Making use of the method, we have successfully dealt with some spherically symmetric nonstationary black holes. The results are consistent with those obtained by calculating the vacuum expectation values of the renormalized energy-momentum tensors. Furthermore, the new method is more exact and more convenient than the old one.

In this paper, we deal with the Hawking effect of a non-spherically symmetric and nonstationary Kinnersley black hole (Kinnersley, 1969). It is impossible to do this by making use of the calculation of the energy-momentum tensors. But it is possible and easy with the new method. Section 2 gives the equation which determines the event horizon of the Kinnersley black hole. The two-dimensional event horizon surface is not spherically symmetric and depends on the time. In Section 3, we introduce a generalized tortoise coordinate transformation and reduce the Klein–Gordon (KG) equation to a

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simple wave equation near the event horizon. In Section 4, both the Hawking temperature and the radiation spectrum are shown. It is very interesting that the Hawking temperature depends on not only the time, but also the angles. Section 5 contains a discussion and conclusion.

2. EVENT HORIZON

The metric of space-time of an arbitrarily accelerating Kinnersley black hole is given by (Kinnersley, 1969)

$$ds^2 = g_{00}\,dv^2 + 2g_{01}\,dv\,dr + 2g_{02}\,dv\,d\theta + 2g_{03}\,dv\,d\varphi + g_{22}\,d\theta^2 + g_{33}d\varphi^2 \tag{1}$$

where

$$\begin{aligned} g_{00} &= 1 - \frac{2M}{r} - 2ar\cos\theta - (b\sin\varphi + c\cos\varphi - a\sin\theta)^2 r^2 \\ &\quad - (b\cos\varphi - c\sin\varphi)^2 r^2\cos^2\theta \\ g_{01} &= -1 \\ g_{02} &= r^2(b\sin\varphi + c\cos\varphi - a\sin\theta) \\ g_{03} &= \sin\theta\cos\theta\,(b\cos\varphi - c\sin\varphi)r^2 \\ g_{22} &= -r^2, \qquad g_{33} = -r^2\sin^2\theta \end{aligned} \tag{2}$$

where $M = M(v)$ is the mass of the black hole, and $a = a(v)$, $b = b(v)$, and $c = c(v)$ are acceleration parameters: a is the magnitude of acceleration and b, c are the rates of change with direction. v is the advanced Eddington coordinate.

It is easy to calculate the metric determinant and its contravariant components, as follows:

$$g = -r^4\sin^2\theta \tag{3}$$

$$\begin{aligned} g^{01} &= -1 \\ g^{11} &= -\left(1 - \frac{2M}{r}\right) + 2ar\cos\theta \\ g^{12} &= -(b\sin\varphi + c\cos\varphi - a\sin\theta) \\ g^{13} &= -(b\cos\varphi - c\sin\varphi)\operatorname{ctg}\theta \\ g^{22} &= -\frac{1}{r^2} \\ g^{33} &= -\frac{1}{r^2\sin^2\theta} \end{aligned} \tag{4}$$

With the aid of the null hypersurface condition

$$g^{\mu\nu} \frac{\partial F}{\partial x^{\mu}} \frac{\partial F}{\partial x^{\nu}} = 0 \tag{5}$$

we infer the event horizon of the space-time described by equation (1). In the above

$$F = F(v, r, \theta, \varphi) = 0 \tag{6}$$

is the hypersurface equation, whose obvious expression is

$$r = r(v, \theta, \varphi) \tag{7}$$

From equations (6) and (7), it is not difficult to get

$$\begin{aligned} &\frac{\partial F}{\partial v} + \frac{\partial F}{\partial r} \frac{\partial r}{\partial v} = 0 \\ &\frac{\partial F}{\partial \theta} + \frac{\partial F}{\partial r} \frac{\partial r}{\partial \theta} = 0 \\ &\frac{\partial F}{\partial \varphi} + \frac{\partial F}{\partial r} \frac{\partial r}{\partial \varphi} = 0 \end{aligned} \tag{8}$$

Substituting (8) into (5), we have

$$g^{11} - 2g^{01}\left(\frac{\partial r}{\partial v}\right) - 2g^{12}\left(\frac{\partial r}{\partial \theta}\right) - 2g^{13}\left(\frac{\partial r}{\partial \varphi}\right) + g^{22}\left(\frac{\partial r}{\partial \theta}\right)^2 + g^{33}\left(\frac{\partial r}{\partial \varphi}\right)^2 = 0 \tag{9}$$

This is just the equation of determining the location of the event horizon. It follows that r_H depends on not only v, but also θ, φ. So the location of the event horizon changes with time, and the shape of the black hole does not keep any symmetry.

3. KLEIN–GORDON EQUATION AND GENERALIZED TORTOISE COORDINATE

The dynamic behavior of the scalar particles is described by the KG equation in curved space-time, namely

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^{\mu}} \left(\sqrt{-g}\, g^{\mu\nu} \frac{\partial \Psi}{\partial x^{\nu}}\right) - \mu_0^2 \Psi = 0 \tag{10}$$

Here, μ_0 is the mass of KG particles. In the space-time (1), equation (10) can be written as

$$g^{11}\frac{\partial^2\Psi}{\partial r^2} - 2\frac{\partial^2\Psi}{\partial v\,\partial r} + 2g^{12}\frac{\partial^2\Psi}{\partial r\,\partial\theta} + 2g^{13}\frac{\partial^2\Psi}{\partial r\,\partial\varphi} + g^{22}\frac{\partial^2\Psi}{\partial\theta^2} + g^{33}\frac{\partial^2\Psi}{\partial\varphi^2}$$

$$+ f_v\frac{\partial\Psi}{\partial v} + f_r\frac{\partial\Psi}{\partial r} + f_\theta\frac{\partial\Psi}{\partial\theta} + f_\varphi\frac{\partial\Psi}{\partial\varphi} - \mu_0^2\Psi = 0 \tag{11}$$

where

$$f_v = -\frac{2}{r}$$

$$f_r = \frac{2}{r}g^{11} + g^{12}\operatorname{ctg}\theta + \frac{\partial g^{11}}{\partial r} + \frac{\partial g^{12}}{\partial\theta} + \frac{\partial g^{13}}{\partial\varphi}$$

$$f_\theta = \frac{2}{r}g^{12} + g^{22}\operatorname{ctg}\theta$$

$$f_\varphi = \frac{2}{r}g^{13} \tag{12}$$

We introduce the generalized tortoise coordinate transformation

$$r_* = r + \frac{1}{2\kappa}\ln[r - r_H(v, \theta, \varphi)]$$

$$v_* = v - v_0, \qquad \theta_* = \theta - \theta_0, \qquad \varphi_* = \varphi - \varphi_0 \tag{13}$$

where r_H is the location of the event horizon, and κ is an adjustable parameter (we will find that κ is the temperature function showing the Hawking radiation of the black hole) and is unchanged under tortoise transformation. v_0, θ_0, φ_0 are all arbitrary constants. From formula (13), we can deduce the following equations:

$$\frac{\partial}{\partial r} = \left[1 + \frac{1}{2\kappa(r - r_H)}\right]\frac{\partial}{\partial r_*}$$

$$\frac{\partial}{\partial v} = \frac{\partial}{\partial v_*} - \frac{1}{2\kappa(r - r_H)}\left(\frac{\partial r_H}{\partial v}\right)\frac{\partial}{\partial r_H}$$

$$\frac{\partial}{\partial\theta} = \frac{\partial}{\partial\theta_*} - \frac{1}{2\kappa(r - r_H)}\left(\frac{\partial r_H}{\partial\theta}\right)\frac{\partial}{\partial r_*}$$

$$\frac{\partial}{\partial\varphi} = \frac{\partial}{\partial\varphi_*} - \frac{1}{2\kappa(r - r_H)}\left(\frac{\partial r_H}{\partial\varphi}\right)\frac{\partial}{\partial r_*}$$

$$\frac{\partial^2}{\partial r^2} = \left[1 + \frac{1}{2\kappa(r - r_H)}\right]^2 \frac{\partial^2}{\partial r_*^2} - \frac{1}{2\kappa(r - r_*)^2} \frac{\partial}{\partial r_*}$$

$$\frac{\partial^2}{\partial \theta^2} = \frac{\partial^2}{\partial \theta_*^2} - \frac{2(\partial r_H/\partial\theta)}{2\kappa(r - r_H)} \frac{\partial^2}{\partial r_* \partial \theta_*} + \left[\frac{(\partial r_H/\partial\theta)}{2\kappa(r - r_H)}\right]^2 \frac{\partial^2}{\partial r_*^2}$$
$$- \frac{(r - r_H)(\partial^2 r_H/\partial\theta^2) + (\partial r_H/\partial\theta)^2}{2\kappa(r - r_H)^2} \frac{\partial}{\partial r_*}$$

$$\frac{\partial^2}{\partial \varphi^2} = \frac{\partial^2}{\partial \varphi_*^2} - \frac{2(\partial r_H/\partial\varphi)}{2\kappa(r - r_H)} \frac{\partial^2}{\partial r_* \partial \varphi_*} + \left[\frac{(\partial r_H/\partial\varphi)}{2\kappa(r - r_H)}\right]^2 \frac{\partial^2}{\partial r_*^2}$$
$$- \frac{(r - r_H)(\partial^2 r_H/\partial\varphi^2) + (\partial r_H/\partial\varphi)^2}{2\kappa(r - r_H)^2} \frac{\partial}{\partial r_*}$$

$$\frac{\partial^2}{\partial v\, \partial r} = \left[1 + \frac{1}{2\kappa(r - r_H)}\right] \frac{\partial^2}{\partial v_* \partial r_*} - \frac{(\partial r_H/\partial v)}{2\kappa(r - r_H)} \left[1 + \frac{1}{2\kappa(r - r_H)}\right] \frac{\partial^2}{\partial r_*^2}$$
$$+ \frac{(\partial r_H/\partial v)}{2\kappa(r - r_H)^2} \frac{\partial}{\partial r_*}$$

$$\frac{\partial^2}{\partial \theta\, \partial r} = \left[1 + \frac{1}{2\kappa(r - r_H)}\right] \frac{\partial^2}{\partial \theta_* \partial r_*} - \frac{(\partial r_H/\partial\theta)}{2\kappa(r - r_H)} \left[1 + \frac{1}{2\kappa(r - r_H)}\right] \frac{\partial^2}{\partial r_*^2}$$
$$+ \frac{(\partial r_H/\partial\theta)}{2\kappa(r - r_H)^2} \frac{\partial}{\partial r_*}$$

$$\frac{\partial^2}{\partial \varphi\, \partial r} = \left[1 + \frac{1}{2\kappa(r - r_H)}\right] \frac{\partial^2}{\partial \varphi_* \partial r_*} - \frac{(\partial r_H/\partial\varphi)}{2\kappa(r - r_H)} \left[1 + \frac{1}{2\kappa(r - r_H)}\right] \frac{\partial^2}{\partial r_*^2}$$
$$+ \frac{(\partial r_H/\partial\varphi)}{2\kappa(r - r_H)^2} \frac{\partial}{\partial r_*} \tag{14}$$

Substituting equation (14) into equation (11), we have

$$\{2\kappa(r - r_H)[1 + 2\kappa(r - r_H)]\}^{-1}\Bigg\{-g^{11}[1 + 2\kappa(r - r_H)]^2$$
$$+ \left[2g^{01}\left(\frac{\partial r_H}{\partial v}\right) + 2g^{12}\left(\frac{\partial r_H}{\partial \theta}\right) + 2g^{13}\left(\frac{\partial r_H}{\partial \varphi}\right)\right][1 + 2\kappa(r - r_H)]$$
$$- \left[g^{22}\left(\frac{\partial r_H}{\partial \theta}\right)^2 + g^{33}\left(\frac{\partial r_H}{\partial \varphi}\right)^2\right]\Bigg\} \frac{\partial^2 \Psi}{\partial r_*^2} + 2\frac{\partial^2 \Psi}{\partial v_* \partial r_*}$$

$$+ [1 + 2\kappa(r - r_H)]^{-1}\left\{\left(2g^{22}\left(\frac{\partial r_H}{\partial\theta}\right) - 2g^{12}[1 + 2\kappa(r - r_H)\right]\right)$$

$$\times \frac{\partial^2\Psi}{\partial\theta_*\partial r_*} + \left(2g^{33}\left(\frac{\partial r_H}{\partial\varphi}\right) - 2g^{13}[1 + 2\kappa(r - r_H)]\right)$$

$$\left.\times \frac{\partial^2\Psi}{\partial\varphi_*\partial r_*} - 2\kappa(r - r_H)g^{22}\frac{\partial^2\Psi}{\partial\theta_*^2} - 2\kappa(r - r_H)g^{33}\frac{\partial^2\Psi}{\partial\varphi_*^2}\right\}$$

$$+ \{(r - r_H)[1 + 2\kappa(r - r_H)]\}^{-1}\left\{g^{11} + 2\left(\frac{\partial r_H}{\partial v}\right)\right.$$

$$- 2g^{12}\left(\frac{\partial r_H}{\partial\theta}\right) - 2g^{13}\left(\frac{\partial r_H}{\partial\varphi}\right) + g^{22}\left[(r - r_H)\left(\frac{\partial^2 r_H}{\partial\theta^2}\right) + \left(\frac{\partial r_H}{\partial\theta}\right)^2\right]$$

$$+ g^{33}\left[(r - r_H)\left(\frac{\partial^2 r_H}{\partial\varphi^2}\right) + \left(\frac{\partial r_H}{\partial\varphi}\right)^2\right] + \left(f_v\left(\frac{\partial r_H}{\partial v}\right) + f_\theta\left(\frac{\partial r_H}{\partial\theta}\right)\right.$$

$$\left.\left. + f_\varphi\left(\frac{\partial r_H}{\partial\varphi}\right) - f_r[1 + 2\kappa(r - r_H)]\right)(r - r_H)\right\}\frac{\partial\Psi}{\partial r_*}$$

$$+ 2\kappa(r - r_H)[1 + 2\kappa(r - r_H)]^{-1}$$

$$\times \left\{-f_\theta\frac{\partial\Psi}{\partial\theta_*} - f_\varphi\frac{\partial\Psi}{\partial\varphi_*} - f_v\frac{\partial\Psi}{\partial v_*} + \mu_0^2\Psi\right\} = 0 \qquad (15)$$

This is the KG equation expressed in tortoise coordinates.

Now, let us deal with the KG equation near the event horizon. When $r \rightarrow r_H(v_0, \theta_0, \varphi_0)$, equation (15) can be reduced to

$$A\frac{\partial^2\Psi}{\partial r_*^2} + 2\frac{\partial^2\Psi}{\partial v_*\partial r_*} + B\frac{\partial^2\Psi}{\partial\theta_*\partial r_*} + C\frac{\partial^2\Psi}{\partial\varphi_*\partial r_*} + D\frac{\partial\Psi}{\partial r_*} = 0 \qquad (16)$$

where

$$A = \lim_{r\rightarrow r_H}\left\{-g^{11}[1 + 2\kappa(r - r_H)]^2 + \left[2g^{01}\left(\frac{\partial r_H}{\partial v}\right) + 2g^{12}\left(\frac{\partial r_H}{\partial\theta}\right)\right.\right.$$

$$\left.\left. + 2g^{13}\left(\frac{\partial r_H}{\partial\varphi}\right)\right][1 + 2\kappa(r - r_H)] - \left[g^{22}\left(\frac{\partial r_H}{\partial\theta}\right)^2 + g^{33}\left(\frac{\partial r_H}{\partial\varphi}\right)^2\right]\right\}$$

$$\times \{2\kappa(r - r_H)[1 + 2\kappa(r - r_H)]\}^{-1}$$

$$B = \lim_{r\to r_H}\left[2g^{22}\left(\frac{\partial r_H}{\partial\theta}\right) - 2g^{12}\right]$$

$$C = \lim_{r\to r_H}\left[2g^{33}\left(\frac{\partial r_H}{\partial\varphi}\right) - 2g^{13}\right]$$

$$D = \lim_{r\to r_H}\left\{\frac{\partial g^{11}}{\partial r} + \left(\frac{\partial g^{22}}{\partial r}\right)\left(\frac{\partial r_H}{\partial\theta}\right)^2 + \left(\frac{\partial g^{33}}{\partial r}\right)\left(\frac{\partial r_H}{\partial\varphi}\right)^2 + g^{22}\left(\frac{\partial^2 r_H}{\partial\theta^2}\right)\right.$$
$$+ g^{33}\left(\frac{\partial^2 r_H}{\partial\varphi^2}\right) + \frac{2}{r}\left[g^{10}\left(\frac{\partial r_H}{\partial v}\right) + g^{12}\left(\frac{\partial r_H}{\partial\theta}\right) + g^{13}\left(\frac{\partial r_H}{\partial\varphi}\right) - g^{11}\right]$$
$$\left. + g^{22}\left(\frac{\partial r_H}{\partial\theta}\right)\operatorname{ctg}\theta - \left(\frac{\partial g^{11}}{\partial r} + g^{12}\operatorname{ctg}\theta + \frac{\partial g^{21}}{\partial\theta} + \frac{\partial g^{31}}{\partial\varphi}\right)\right\}$$

Obviously, A, B, C, and D will all be regarded as finite constants in equation (16). But, unlike B, C, and D, with the null-surface equation (5), we know that

$$\lim_{r\to r_H}\left\{-g^{11}[1 + 2\kappa(r - r_H)]^2 + \left[2g^{01}\left(\frac{\partial r_H}{\partial v}\right) + 2g^{12}\left(\frac{\partial r_H}{\partial\theta}\right)\right.\right.$$
$$\left.\left. + 2g^{13}\left(\frac{\partial r_H}{\partial\varphi}\right)\right][1 + 2\kappa(r - r_H)] - \left[g^{22}\left(\frac{\partial r_H}{\partial\theta}\right)^2 + g^{33}\left(\frac{\partial r_H}{\partial\varphi}\right)^2\right]\right\} = 0 \qquad (17)$$

so the limit A is 0/0, an indefinite form. By use of the L'Hôspital rule, we obtain the following result:

$$A = \left\{-\frac{\partial g^{11}}{\partial r} - 4\kappa g^{11} + 2\kappa\left[2g^{01}\left(\frac{\partial r_H}{\partial v}\right) + 2g^{12}\left(\frac{\partial r_H}{\partial\theta}\right) + 2g^{13}\left(\frac{\partial r_H}{\partial\varphi}\right)\right]\right.$$
$$\left. - \frac{\partial g^{22}}{\partial r}\left(\frac{\partial r_H}{\partial\theta}\right)^2 - \frac{\partial g^{33}}{\partial r}\left(\frac{\partial r_H}{\partial\varphi}\right)^2\right\}$$
$$\times (2\kappa)^{-1}\Big|(r_H, v_0, \theta_0, \varphi_0) \qquad (18)$$

Selecting the adjustable parameter κ in equation (13) as

$$\kappa = \left[-\frac{\partial g^{11}}{\partial r} - \frac{\partial g^{22}}{\partial r}\left(\frac{\partial r_H}{\partial\theta}\right)^2 - \frac{\partial g^{33}}{\partial r}\left(\frac{\partial r_H}{\partial\varphi}\right)^2\right]$$
$$\times\left[2\left(1 + 2g^{11} - 2g^{01}\frac{\partial r_H}{\partial v} - 2g^{12}\frac{\partial r_H}{\partial\theta} - 2g^{13}\frac{\partial r_H}{\partial\varphi}\right)\right]^{-1}\Bigg|_{(r_H, v_0, \theta_0, \varphi_0)} \qquad (19)$$

then we have $A = 1$.

The KG equation has been reduced to a simple wave equation.

4. THE HAWKING THERMAL SPECTRUM

Separating variables as follows

$$\Psi(v_*, r_*, \theta_*, \varphi_*) = e^{-i\omega v_*}R(r_*)\Theta(\theta_*)\Phi(\varphi_*) \tag{20}$$

and substituting this into equation (16), we have

$$\frac{R''}{R'} + (D - 2i\omega) = -\left(B\frac{\Theta'}{\Theta} + C\frac{\Phi'}{\Phi}\right) \tag{21}$$

It can be seen that both sides are equal to the same complex constant number, which is $-(\lambda_0 + 2i\omega_0)$, and then

$$R'' + [D + \lambda_0 - 2i(\omega - \omega_0)]R' = 0 \tag{22}$$

$$B\frac{\Theta'}{\Theta} + C\frac{\Phi'}{\Phi} = \lambda_0 + 2i\omega_0 \tag{23}$$

Because for radiation only the radial equation is relevant, we are not interested in equation (23). The solution of equation (22) is

$$\Psi_\omega^{\text{in}} \sim e^{-i\omega v_*} \tag{24}$$

$$\Psi_\omega^{\text{out}} \sim e^{-i\omega v_*}e^{2i(\omega-\omega_0)r_*}e^{-(D+\lambda_0)r_*} \tag{25}$$

Near the event horizon, we have

$$r_* \sim \frac{1}{2\kappa}\ln(r - r_H) \tag{26}$$

and equation (25) can be written as

$$\Psi_\omega^{\text{out}} \sim e^{-i\omega v_*}(r - r_H)^{i(\omega-\omega_0)/\kappa}(r - r_H)^{-(D+\lambda_0)/2\kappa} \tag{27}$$

It is clear that Ψ_ω^{out} is not analytical at $r = r_H$, so we have to analytically extend it through the lower half complex r plane into the inside of the event horizon and obtain

$$|r - r_H| \to |r - r_H|^{e^{-i\pi}} = (r_H - r)e^{-i\pi} \tag{28}$$

So we have

$$\begin{aligned}\tilde{\Psi}_\omega^{\text{out}}(r < r_H) &\sim e^{-i\omega v_*}[(r_H - r)e^{-i\pi}]^{i(\omega-\omega_0)/\kappa}[(r_H - r)e^{-i\pi}]^{-(D+\lambda_0)/2\kappa} \\ &= e^{-i\omega v_*}e^{2i(\omega-\omega_0)r_*}e^{-(D+\lambda_0)r_*}e^{i(D+\lambda_0)\pi/2\kappa}e^{(\omega-\omega_0)\pi/\kappa}\end{aligned} \tag{29}$$

The scattering probability of the outgoing wave at the horizon is

$$\left|\frac{\Psi_{\omega}^{\text{out}}(r > r_H)}{\tilde{\Psi}_{\omega}^{\text{out}}(r < r_H)}\right|^2 = e^{-2\pi(\omega-\omega_0)/\kappa} \tag{30}$$

According to the method suggested by Damour and Ruffini (1976) and Sannan (1988), the Hawking thermal spectrum is given by

$$N_{\omega} = \frac{1}{\exp[(\omega - \omega_0)/K_B T] - 1} \tag{31}$$

where K_B is Boltzmann's constant. The Hawking temperature is given by

$$T = \frac{\kappa}{2\pi K_B} \tag{32}$$

It follows that κ is the function determining the Hawking temperature. The temperature depends not only on the time, but also the angles θ and φ.

5. CONCLUSION AND DISCUSSION

When $a = b = c = 0$, the metric (1) is reduced to Vaidya metric (Balbinot, 1986)

$$ds^2 = \left(1 - \frac{2M}{r}\right) dv^2 - 2\, dv\, dr - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\varphi^2 \tag{33}$$

From equations (9) and (19) we have

$$r_H = \frac{2M}{1 - 2\dot{r}_H} \approx 2M(1 + 4\dot{M}), \qquad T = \frac{\kappa}{2\pi K_B}$$

$$\kappa = \frac{1 - 2\dot{r}_H}{4M} \approx \frac{1 - 4\dot{M}}{4M} \tag{34}$$

These are just the well-known event horizon and temperature of the Vaidya black hole.

We have studied the Hawking radiation of the Kinnersley black hole whose mass changes with time. Both the location of the event horizon and the Hawking temperature depend on the time and the angles.

We emphasize that there is a conspicuous absence of any report up to now on the fact that the Hawking temperature changes with angles, for which our research has supplied an example where the Hawking temperature at a point differs from another point on the two-dimensional simultaneity surface of the event horizon.

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